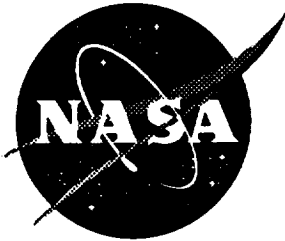


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# Dynamic Fracture Mechanics Analysis for an Edge Delamination Crack

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## ABSTRACT

A global/local analysis is applied to the problem of a panel with an edge delamination crack subject to an impulse loading to ascertain the dynamic  $J$  integral. The approach uses the spectral element method to obtain the global dynamic response and local resultants to obtain the  $J$  integral. The variation of  $J$  integral along the crack front is shown. The crack behavior is mixed mode (Mode II and Mode III), but is dominated by the Mode II behavior.

## INTRODUCTION

Integrally stiffened panels, such as those shown in Figure 1, are commonly used in aircraft structural design. Static pressure loads have been found to cause high stresses in the vicinity of the flange termination line which can cause stiffener debonding from the skin [1]. Fracture mechanics analyses have previously been performed on these structures [2, 3] to determine the static strain energy release rate using two- and three-dimensional finite element analyses. Static pressure loads are not the only mechanism for skin-stiffener debonding, however. A recent series of acoustic tests on stiffened carbon/carbon panels [4, 5] indicate that dynamic pressure loading can produce similar failures. Because of resonant peaks in the structural response, the dynamic loads need not be as large as the static loads, particularly for lightly damped structures. Thus, the dynamic, and not the static loads, may ultimately be the cause of failure in these structures. This paper presents an approach for determining the dynamic fracture mechanics quantities. To help focus ideas, the problem of an edge delamination crack, as shown in Figure 1, is analyzed to illuminate the features of the local crack dynamics. The delamination is taken as being symmetric about the  $z = 0$  plane.

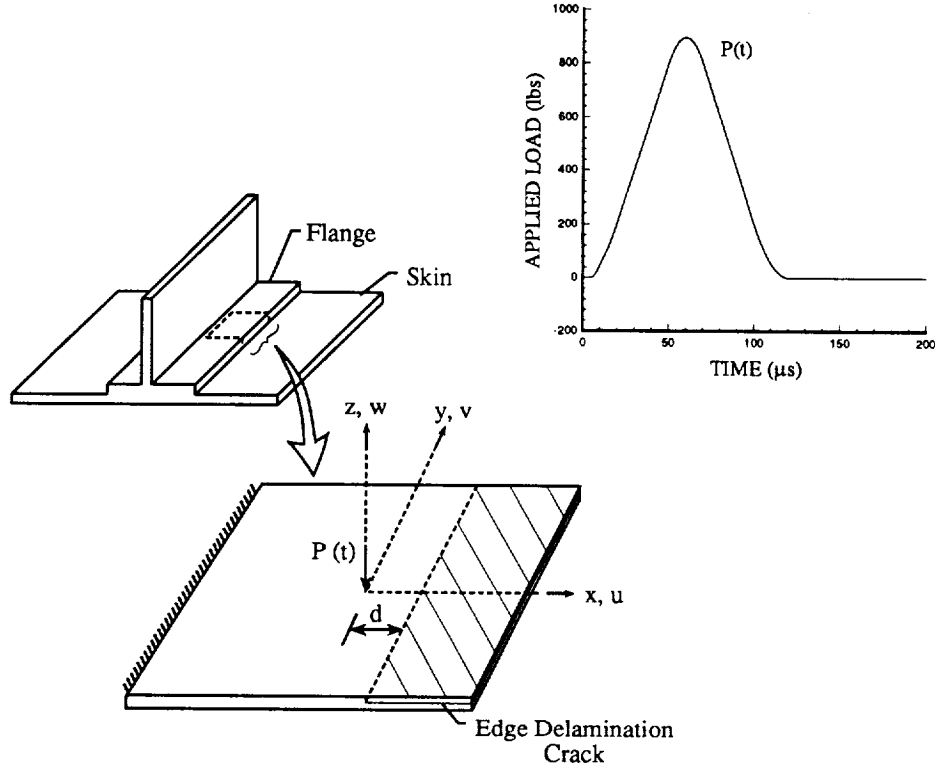


Figure 1: Section of an integrally stiffened panel aircraft panel.

The fracture mechanics approach previously employed, based on two-dimensional plate finite elements [2] or three-dimensional brick finite elements [3] is unsuitable for the dynamics problem; the necessity to adequately distribute the mass requires a much finer finite element model than one used to model the structure for the static problem, even in areas remote from the crack tip. Therefore, the use of alternative methods is indicated.

The objective of this paper is to develop an approach to determine the dynamic  $J$  integral [6, 7]. The  $J$  integral, in the Mode II “shearing mode” for example, is related to the strain energy release rate  $G$  and the stress intensity factor  $K$  as

$$J = G = \frac{K_{II}^2}{E} \quad (1)$$

and therefore is an appropriate measure of the local crack tip behavior. (In equation (1),  $E$  is the modulus of elasticity). Although the quantities used in the dynamic  $J$  integral could be determined from a conventional finite element analysis, this approach is not taken because of the large memory requirements. Instead, the dynamic analysis is performed using the spectral element method [8]. Unlike the finite element method, the spectral element formulation models the mass distribution exactly, and consequently, only a very few elements are required, resulting in a greatly reduced system size. The spectral element used is an augmentation of an in-plane spectral element [9] which includes the flexural degrees of freedom.

Comparisons are first made between the spectral element analysis and a three-dimensional plate finite element analysis to establish the adequacy of the spectral element model of the global dynamics of the cracked panel. Results from a plane strain dynamic analysis are compared to establish the validity of the fracture mechanics approach used. Finally,  $J$  integrals are presented for stress waves with general, non-plane distributions. The variation of the quantities along the crack front are discussed.

## DYNAMIC FRACTURE MECHANICS ANALYSIS

A global/local approach is taken to model the crack dynamics. That is, the problem is separated into the global structural dynamics of connected three-dimensional plates on the one hand, and the local crack tip behavior on the other. This approach has proven to be very accurate and convenient in the case of two-dimensional split beam problems [10, 11] and, therefore, is a natural approach for the complicated three-dimensional plate problem of interest here.

### Global Dynamics

The specific problem studied is that of a thin sheet containing an edge delamination crack, as shown in Figure 1. Isotropic material properties were used in the present analysis to help concentrate attention on the application of the approach. The analysis of composite structures will be left as an item for further study. The material properties used were for aluminum: Young's Modulus =  $10.6 \times 10^6$  psi, shear modulus =  $4.0 \times 10^6$  psi, and density =  $2.61 \times 10^{-4}$  lb-s<sup>2</sup>/in<sup>4</sup>. The plate thickness was taken as 0.065 in. Each leg of the cracked region was 3 in. long. An impulse loading,  $P(t)$ , applied at a distance  $d = 3$  in. from the crack, is shown in Figure 1.

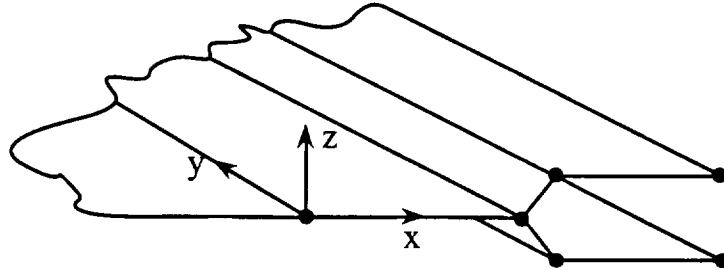


Figure 2: Spectral element discretization of an edge delamination crack.

The cracked panel was modeled as a three-dimensional structure of connected flat plates using spectral elements, as shown in Figure 2. The black dots indicate generalized nodes that extend indefinitely in the  $y$ -direction. An advantage of the spectral approach was that the material in the negative  $x$ -direction could be modeled using a semi-infinite plate. This allowed the responses from the crack to be focused

on without reflections off the far boundary. Each spectral element used can support both in-plane [9] and out-of-plane (flexural) [8] motions. The initial disturbance generated was flexural only, but on reaching the crack, in-plane behavior results because of the vertical offset of the split plate centerlines in the cracked region. The result of the global analysis is the structural degrees of freedom (displacements and rotation)

$$u, v, w, \phi_y$$

and structural resultants (forces and moment) per unit length

$$N_x, N_y, N_z, M_y$$

at any location and time. A double summation of 512 wavenumber and 2048 frequency components was used for a spatial window of 400 in. and time window of 14,329  $\mu s$ .

For the purpose of verifying the global dynamics, a three-dimensional plate finite element analysis was also performed. This used DKT elements [12] to model the out-of-plane behavior and constant strain triangle elements [13] for the in-plane behavior. The model extended 6 in. toward the negative  $x$ -direction and 9 in. toward the positive  $y$ -direction, with the  $x$ -axis taken as a line of symmetry. A uniform mesh of 8640 elements, each with a characteristic length of 0.25 in., was used and the system was solved using Newmark integration at 1  $\mu s$  increments. Assembly of multiply connected plate structures produces large excursions from the mean bandwidth of the system, so a profile solver [13] was implemented in the finite element program so that the system could be solved.

Comparisons of the velocity histories are shown in Figure 3 for several locations along the  $x$ -axis. The spectral and finite element analyses agree well up to the point of reflections off the negative  $x$  and positive  $y$  boundaries in the finite element model. The vertical  $w$  velocities of the top and bottom split sections are identical, indicating the absence of Mode I “opening mode” crack behavior. This is a consequence of symmetry about the crack ( $z = 0$ ) plane and that the delaminated regions are equal in length above and below the crack plane. Note however that a longer section on one side of the crack would additionally generate Mode I behavior because the non-symmetric reflections would cause a non-symmetric loading at the crack tip. Of particular note is the presence of an in-plane velocity component in the delaminated area. The horizontal  $u$  velocities of the top and bottom split sections are equal and opposite indicating the presence of Mode II “sliding mode” crack behavior.

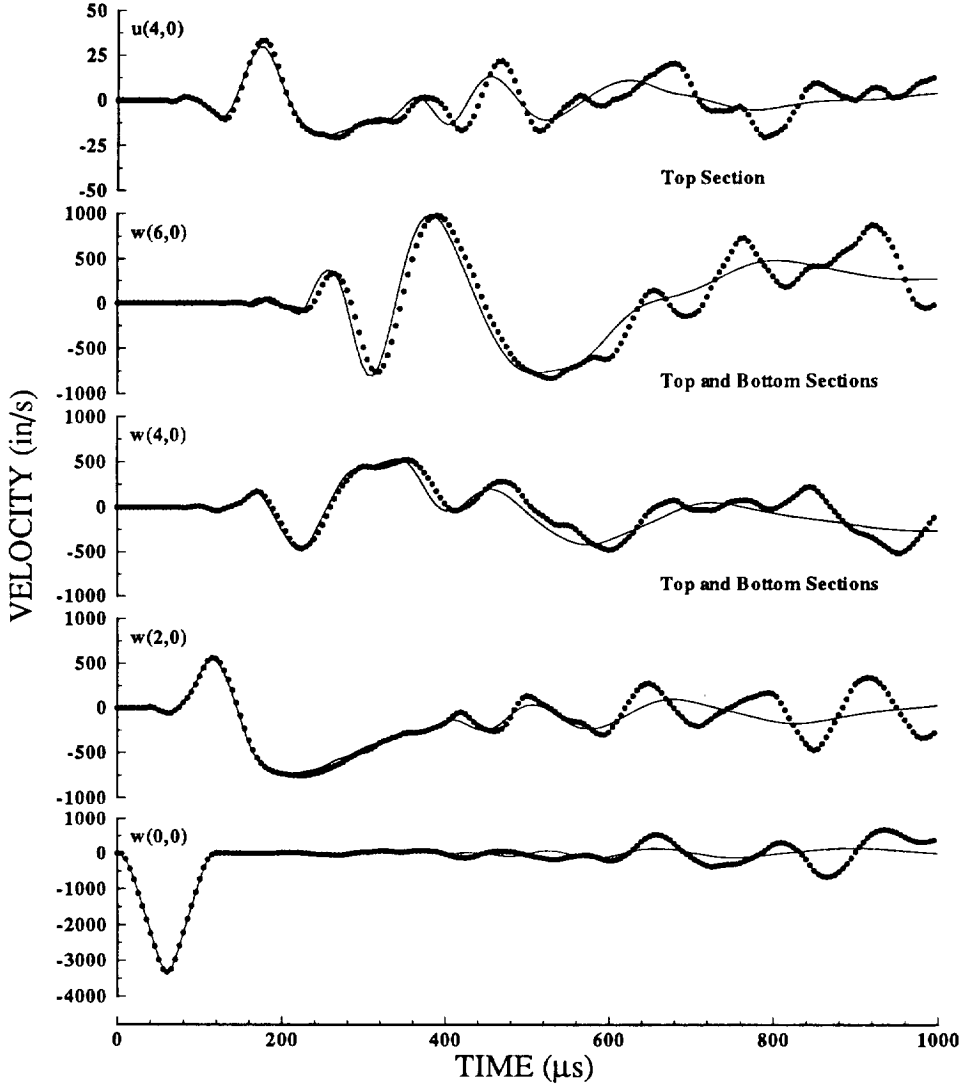


Figure 3: Comparison of velocities obtained from spectral (—) and finite element (···) methods for the split plate.

### Local Fracture Mechanics

The dynamic  $J$  integral is related to the local stresses and displacements by [14]

$$J = \int_{\Gamma} \left[ W n_1 - t_i \frac{\partial u_i}{\partial x_1} \right] ds + \int_V \left[ \rho \ddot{u}_i \frac{\partial u_i}{\partial x_1} \right] dV \quad (4)$$

where  $\Gamma$  is a line contour around the crack,  $V$  is the enclosed volume per unit length,  $W$  is the strain energy density,  $n_1$  is the unit normal vector pointing outside the integration path  $\Gamma$ ,  $t_i$  is the traction vector, and  $u_i$  is the displacement.

The volume integral usually causes computational difficulties because it requires the evolution of the acceleration at every point inside the volume and must be performed at each time step. We avoid this difficulty by collapsing the inclined member model of the local crack region used in the previous section, to one using vertical members with reduced density. This results in a zero-volume contour as shown in Figure 4 which encompasses only the cross-sectional discontinuity at the crack tip. On a continuous system, this would give a trivial zero integral, but if we first replace the stresses and strains in terms of the structural resultants we get

$$2EJ = - \sum h n_x \left[ \frac{N_x^2}{h^2} + \frac{12(M_x^2 + M_y^2)}{h^4} - 2.4(1 + \nu) \frac{N_z^2}{h^2} + 2E \frac{N_z \phi_y}{h} \right] \quad (5)$$

where the summation is over each section of the crack,  $\nu$  is the Poisson's ratio, and  $h$  is the plate thickness. The local analysis has thus taken the global resultants and converted them into detailed information (via  $J$ ) about the crack tip. It is worth pointing out that the use of vertical spectral members has little effect on the predicted global dynamics in comparison to results from the inclined spectral model. The inclined model was used solely for comparison with the finite element model, which can not accommodate the high aspect ratio elements required to model the vertical members.

### Method Validation

To help validate the approach taken, a comparison was made for the plane strain case. For this case, the load was distributed uniformly in the  $y$ -direction giving only  $x$  and  $t$  variations in the responses. The spectral analysis used a single (plane wave) wavenumber component. Results from a two-dimensional plane strain finite element analysis employing the virtual crack closure technique (VCCT) [15] were used for comparison. Using the VCCT, the stress intensity was obtained as

$$K_{II} = \sqrt{\frac{(u_t - u_b) F_x E (1 - \nu^2)}{2b\Delta a}} \quad (6)$$

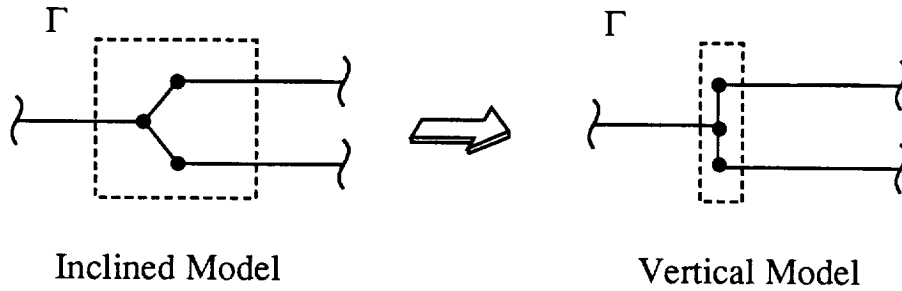


Figure 4: Zero volume  $J$  integral path showing the use of vertical spectral elements.



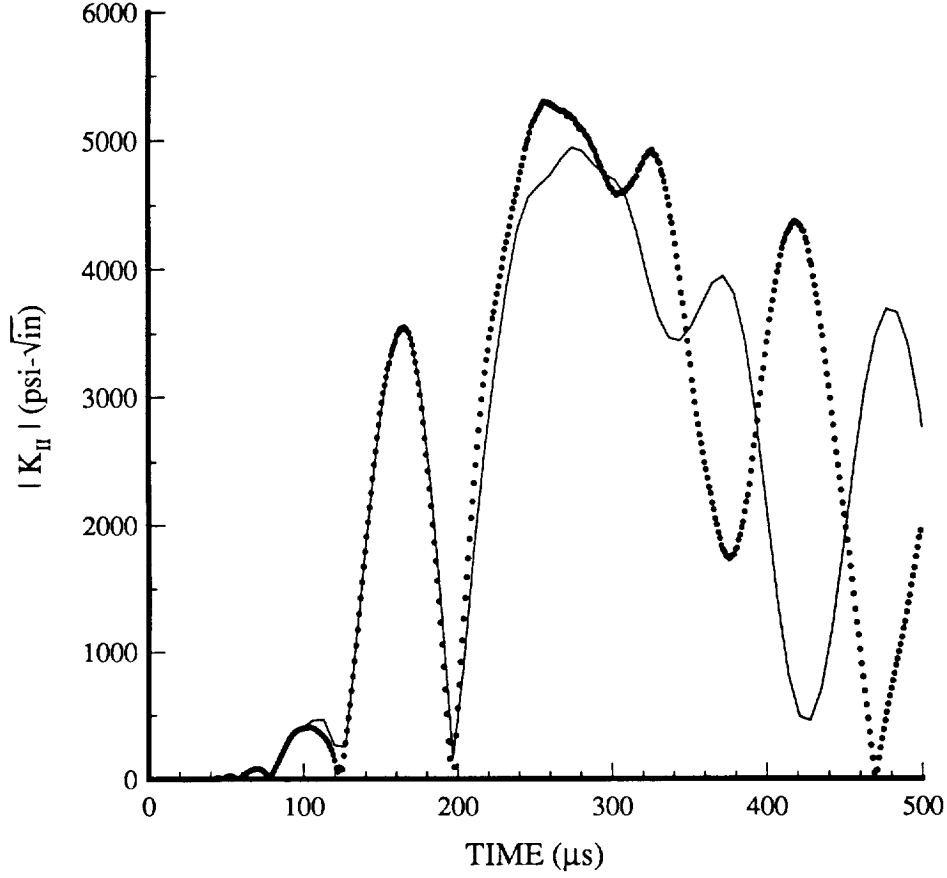


Figure 5: Comparison of  $K_{II}$  from spectral (—) and finite element (···) methods for the plane strain problem.

where  $u$  is the top and bottom horizontal displacement along the crack one node from the crack tip,  $F_x$  is the horizontal nodal force at the crack tip,  $b$  is the thickness in the  $y$ -direction, and  $\Delta a$  is the element size. Figure 5 shows a comparison of the magnitude of the Mode II stress intensity factor as a function of time. The two solutions agree to the extent of the reflections in the finite element model. These results establish the basic validity of our global/local approach.

## RESULTS AND DISCUSSION

$J$  integral histories were obtained for the general, non-plane wave, case using the global/local approach described. As previously indicated, the Mode I contribution for this problem is zero since both upper and lower sections of the delaminated area have the same displacement. The variation of  $J$  integral along the crack front

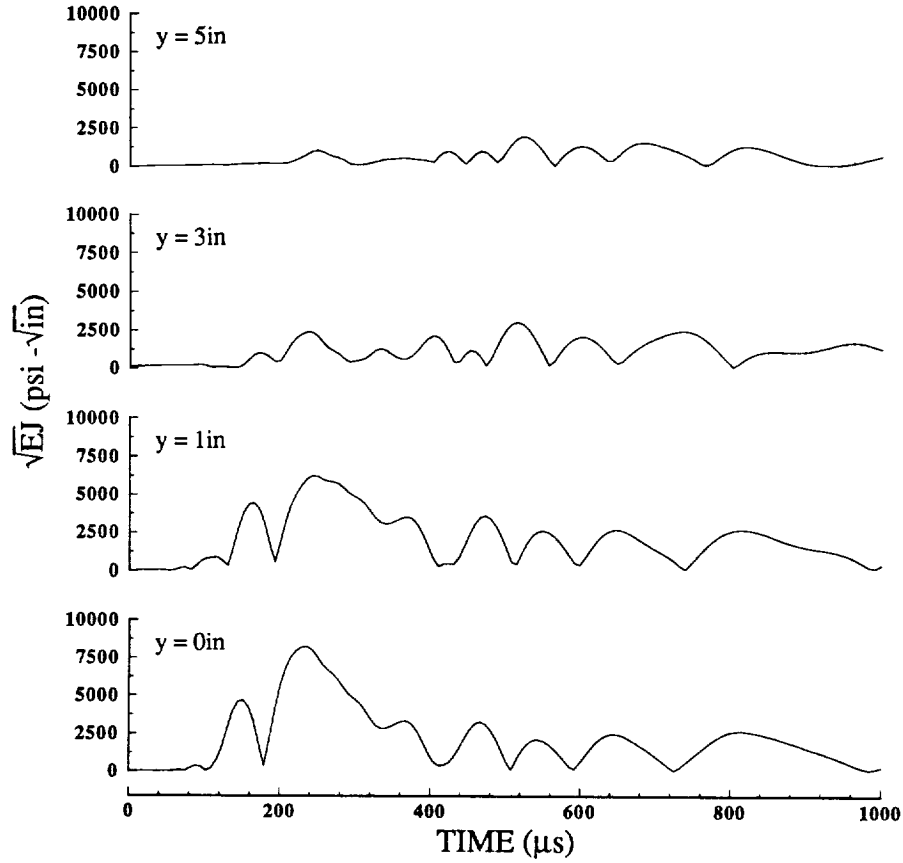


Figure 6:  $J$  integral histories as a function of  $y$  for a panel with an edge delamination crack.

is shown in Figure 6. The magnitudes decrease with increasing distance from the loading. At  $y = 0$ , the plot corresponds to a pure Mode II situation. Further, since the  $J$  integral is a maximum at  $y = 0$ , Figure 6 indicates the problem is dominated by the Mode II behavior.

At non-zero  $y$ , there is also a Mode III “tearing mode” contribution. Figure 7 shows the difference in the horizontal  $v$  displacement at non-zero  $y$ , indicating the presence of a Mode III component. This difference increases to a maximum then decreases with increasing  $y$ . This can be explained by realizing that as  $y$  increases, the normal to the wave front becomes increasingly parallel to the crack front, so the grazing incidence wave causes rotation about an axis parallel to the  $x$ -axis. This effect is not observed in the plane strain case as the normal to the wave front is always perpendicular to the crack front. The presence of the Mode III behavior is interesting and is worthy of further investigation.

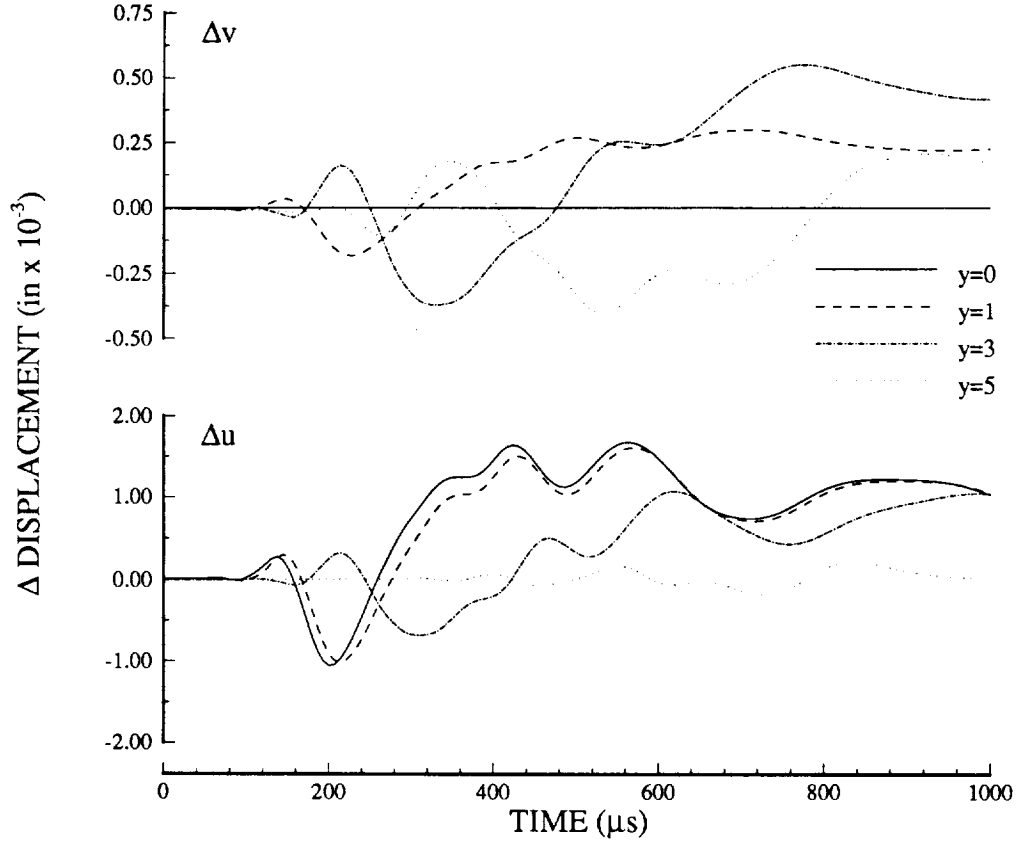


Figure 7: Change in displacement of horizontal  $u$  and  $v$  components.

### CONCLUDING REMARKS

A global/local analysis has been applied to obtain the  $J$  integral along the crack front of a panel with an edge delamination. The approach is computationally attractive in comparison to the finite element approach since the system size is greatly reduced. Comparison of the results with those obtained from a finite element analysis for the plane strain case validates the application of the global/local approach to the class of problems considered.

The application of this approach to the delamination problem has provided an interesting view of the dynamic crack behavior. The Mode II behavior is dominant and varies significantly along the crack front, being a maximum at  $y = 0$ . Since the geometry considered was symmetric about the crack plane, the Mode I behavior is absent. In the case of the integrally stiffened panel, however, symmetry about the crack plane does not exist and the crack is likely to additionally exhibit a Mode I behavior. This aspect is left as an area for further study. With the presence of a Mode III behavior established, this study indicates it is necessary

to model the problem in a three-dimensional manner for the general, non-plane wave, case. As it is not practical to do so using a dynamic three-dimensional brick finite element analysis, the use of an alternative method, such as the global/local analysis presented here, is dictated. It remains to be seen how the Mode II and III contributions can be quantitatively separated through the use of the global/local analysis.

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